



Grade 9/10 Math Circles

March 29, 2023

An Introduction to Group Theory Part 2 - Problem Set

1. Compute the following compositions:

(a) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 2 & 1 & 6 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 6 & 4 \end{pmatrix}$

2. Compute the inverses of the following permutations:

(a) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 2 & 1 & 6 & 4 \end{pmatrix}$

3. Let P_n be a regular polygon with n sides where $n \geq 4$. Convince yourself that the symmetry group of P_n is not the same as the symmetric group on $\{1, \dots, n\}$.

Hint: how many elements are in $Sym(P_n)$ and S_n ?